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5-3 SIMILAR POLYGONS

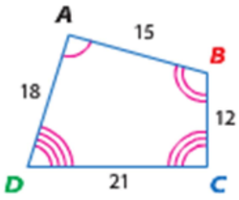
1 Identify Similar Polygons

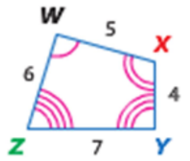
Similar polygons have the same shape but not necessarily the same size.

KeyConcept Similar Polygons

Two polygons are similar if and only if their corresponding angles are congruent and corresponding side lengths are proportional.

Example In the diagram below, *ABCD* is similar to *WXYZ*.





Corresponding angles

$\angle A \cong \angle W$, $\angle B \cong \angle X$, $\angle C \cong \angle Y$, and $\angle D \cong \angle Z$

Corresponding sides

$\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW} = \frac{3}{1}$

$\frac{15}{5} = \frac{12}{4} = \frac{21}{7} = \frac{18}{6} = \frac{3}{1}$

scale factor

sim. ratio

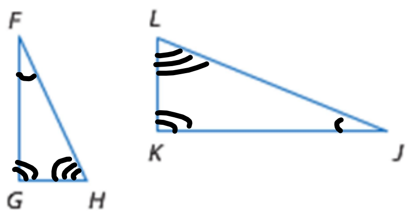
Symbols *ABCD* ~ *WXYZ*

As with congruence statements, the order of vertices in a similarity statement like *ABCD* ~ *WXYZ* is important. It identifies the corresponding angles and sides.

~ = similar

Example 1 Use a Similarity Statement

If △*FGH* ~ △*JKL*, list all pairs of congruent angles, and write a proportion that relates the corresponding sides.



Use the similarity statement.

△*FGH* ~ △*JKL*

Congruent angles: $\angle F \cong \angle J$, $\angle G \cong \angle K$, $\angle H \cong \angle L$

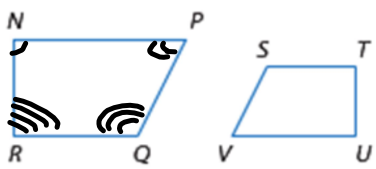
Proportion: $\frac{FG}{JK} = \frac{GH}{KL} = \frac{HF}{LJ}$ = equivalent ratios (same value)

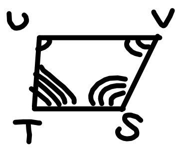
EXAMPLE 1: In the diagram, *NPQR* ~ *UVST*. List all pairs of congruent angles, and write a proportion that relates the corresponding sides.

$\angle N \cong \angle U$
$\angle P \cong \angle V$
$\angle Q \cong \angle S$
$\angle R \cong \angle T$
$\cong \angle S$

$\frac{NP}{UV} = \frac{PQ}{VS} = \frac{QR}{ST} = \frac{RN}{TU}$

sides proportional





scale factor = $\frac{\text{image}}{\text{preimage}}$

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The ratio of the lengths of the corresponding sides of two similar polygons is called the **similarity ratio** (often referred to as the **scale factor**). The similarity ratio depends on the order of comparison.

In the diagram, $\triangle ABC \sim \triangle XYZ$.

The scale factor of $\triangle ABC$ to $\triangle XYZ$ is $\frac{6}{3}$ or 2.

The scale factor of $\triangle XYZ$ to $\triangle ABC$ is $\frac{3}{6}$ or $\frac{1}{2}$.

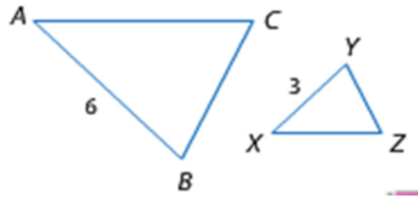

PHOTO EDITING Kuma wants to use the rectangular photo shown as the background for her computer's desktop, but she needs to resize it. Determine whether the following rectangular images are similar. If so, write the similarity statement and scale factor. Explain your reasoning.

preimage

a. possible image

b. possible image

$\frac{I}{P}$ $\frac{FG}{BC}$ $\frac{12}{8} = \frac{3}{2}$

$\frac{HG}{DC} = \frac{14}{10} = \frac{7}{5}$

not same

not similar because the sides are not proportional.

$\frac{I}{P}$ $\frac{KL}{BC}$ $\frac{12}{8} = \frac{3}{2}$

$\frac{ML}{DC}$ $\frac{15}{10} = \frac{3}{2}$

same

Similar because the sides ARE proportional.

ABCD ~ JKLM and the scale factor is $\frac{3}{2}$.

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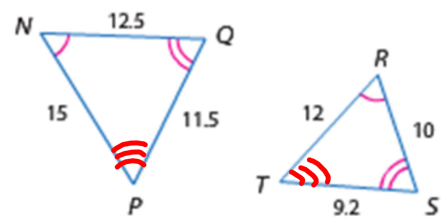
EXAMPLE 2:

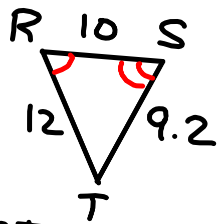
Determine whether the triangles shown are similar. If so, write the similarity statement and scale factor. Explain your reasoning.



$\frac{NQ}{RS}$ $\frac{12.5}{10} = 1.25$

$\frac{QP}{ST}$ $\frac{11.5}{9.2} = 1.25$ } Same

$\frac{PN}{TR}$ $\frac{15}{12} = 1.25$

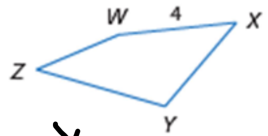


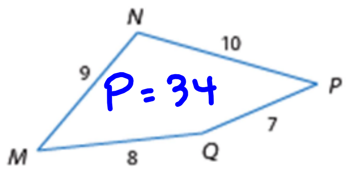
$\triangle NQP \sim \triangle RST \longrightarrow$ scale factor = 1.25

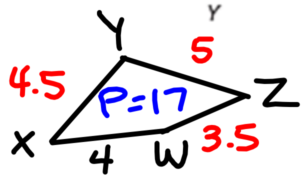
similarity statement

EXAMPLE 3: know

If MNPQ ~ XYZW, find the similarity ratio of MNPQ (Top) to XYZW (Bottom), find the lengths of all the missing sides of XYZW, and find the perimeters of both figures.







$\frac{MNPQ}{XYZW}$ $\frac{MQ}{XW} = \frac{8}{4} = \frac{2}{1}$

Sim. ratio = $\frac{2}{1}$

$\frac{MN}{XY}$ $\frac{9}{XY} = \frac{2}{1}$ $2XY = 9$, $XY = 4.5$

$\frac{NP}{YZ}$ $\frac{10}{YZ} = \frac{2}{1}$ $2YZ = 10$, $YZ = 5$

$\frac{PQ}{ZW}$ $\frac{7}{ZW} = \frac{2}{1}$ $2ZW = 7$, $ZW = 3.5$

Now, compare the perimeters of MNPQ to XYZW. What did you notice? ______

The Perimeter Ratio = the Similarity ratio.

$\frac{P \text{ of } MNPQ}{P \text{ of } XYZW}$ $\frac{34}{17} = \frac{2}{1}$

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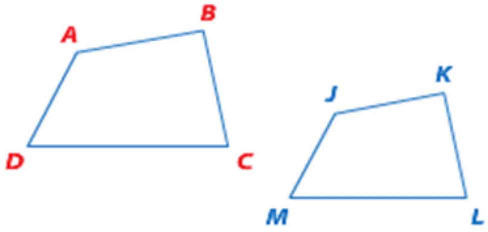
Theorem 7.1 Perimeters of Similar Polygons

If two polygons are similar, then their perimeters are proportional to the scale factor between them.

Example If $ABCD \sim JKLM$, then

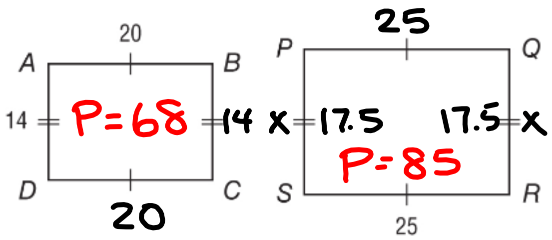
$$\frac{AB+BC+CD+DA}{JK+KL+LM+MJ} = \frac{AB}{JK} = \frac{BC}{KL} = \frac{CD}{LM} = \frac{DA}{MJ}$$



Perimeter Ratio = Similarity Ratio = Scale Factor.

EXAMPLE 4:

If $ABCD \sim PQRS$, find the scale factor of ABCD to PQRS and the perimeter of each polygon.



$\frac{ABCD}{PQRS}$ $\frac{DC}{SR}$ or $\frac{AB}{PQ}$ $\frac{20}{25} = \frac{4}{5}$

scale factor = $\frac{4}{5}$ or 0.8

$\frac{AD}{PS}$ $\frac{14}{x} = \frac{4}{5}$

$4x = 14(5)$

$\frac{4x}{4} = \frac{70}{4}$

$x = 17.5$

Check: $\frac{\text{P of ABCD}}{\text{P of PQRS}}$ $\frac{68}{85} = 0.8$ ✓